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The limits of θ are $\frac{1}{4}\pi$ and $\frac{1}{4}\pi+\frac{1}{2}\beta=\theta_1$.

$$\therefore \triangle=\frac{2}{\beta}\int_{\frac{1}{4}\pi}^{\theta_1} x^2\,d\,\theta=\text{average area.}$$

$$\therefore \triangle=\frac{4r^2\sin^2\beta}{\beta(c-1)}\left[\tan^{-1}\left(\frac{c-\sqrt{(2c-1)}}{c-1}\tan\left(\tfrac{1}{4}\pi+\tfrac{1}{2}\beta-\tfrac{1}{2}\alpha\right)\right)\right.$$

$$\left.-\tan^{-1}\left(\frac{c-\sqrt{(2c-1)}}{c-1}\tan\left(\tfrac{1}{4}\pi-\tfrac{1}{2}\alpha\right)\right)\right],$$

where $c=2+\sin2\beta-\cos2\beta$, $1+\sin2\beta=\sqrt{(2c-1)}\sin\alpha$,
$\cos2\beta-1=\sqrt{(2c-1)}\cos\alpha$.

If $\beta=\frac{1}{4}\pi$, $\triangle=\frac{4r^2}{\pi}\{\tan^{-1}\frac{1}{2}[\sqrt{10}-3][3-\sqrt{5}]+\tan^{-1}\frac{1}{2}[3-\sqrt{5}][\sqrt{5}-2]\}$.

$$\therefore \triangle=.1933r^2.$$

If we accept the usual definition of inscription of geometric figures, viz., the vertices of the inscribed figure shall lie on the boundary of the other figure, the above problem is impossible; for a square cannot be inscribed in a sector so that its diagonal shall be parallel to a random direction. For a sector less than a quadrant three squares can be inscribed, and only three. The average area would be one-third of the sum of the areas of these three squares. The above solution satisfies the case when three vertices of the square lie on the boundary of the sector. The author of the problem may have wished the solution to cover the case when the vertices lie wholly within the sector, the limiting case being when three vertices lie on the boundary. In this case he should not have used the term "inscribed." Ed. F.

MISCELLANEOUS.

172. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

If ϕ and ψ are small angles, show that an approximate value of ϕ/ψ is

$$\tfrac{2}{3}\frac{\sin\phi}{\sin\psi}+\tfrac{1}{3}\frac{\tan\phi}{\tan\psi}-\tfrac{1}{180}(\phi^2-\psi^2)(9\phi^2-\psi^2).$$

II. Solution by the PROPOSER.

Neither Prof. Zerr nor Mr. Greenwood seems to have carried the expansion quite far enough. When this is done the approximation stated will be found to be perfectly correct. The following relations will show this to be the case.

$$\tfrac{2}{3}\frac{\sin\phi}{\sin\psi}=\tfrac{2}{3}\left[\phi-\frac{\phi^3}{6}+\frac{\phi^5}{120}\right]\left[\psi-\frac{\psi^3}{6}+\frac{\psi^5}{120}\right]^{-1}\text{, approximately,}$$

$$=\tfrac{2}{3}\frac{\phi}{\psi}\left[1-\left(\frac{\phi^2}{6}-\frac{\phi^4}{120}\right)\right]\left[1-\left(\frac{\psi^2}{6}-\frac{\psi^4}{120}\right)\right]^{-1}\cdots$$

$$=\tfrac{2}{3}\frac{\phi}{\psi}\left[1+\frac{\psi^2}{6}-\frac{\phi^2}{6}+\frac{\psi^4}{36}-\frac{\psi^4}{120}+\frac{\phi^4}{120}-\frac{\phi^2\,\psi^2}{36}\right]\cdots$$

$$\text{So, }\tfrac{1}{3}\frac{\tan\phi}{\tan\psi}=\tfrac{1}{3}\frac{\phi}{\psi}\left[1+\left(\frac{\phi^2}{3}+\frac{2\phi^4}{15}\right)\right]\left[1+\left(\frac{\psi^2}{3}+\frac{2\psi^4}{15}\right)\right]^{-1}\cdots$$

$$=\tfrac{1}{3}\frac{\phi}{\psi}\left[1+\frac{\phi^2}{3}-\frac{\psi^2}{3}-\frac{2\psi^2}{15}+\frac{\psi^4}{9}+\frac{2\phi^4}{15}-\frac{\phi^2\,\psi^2}{9}\right]\cdots$$

$$\text{Their sum}=\frac{\phi}{\psi}\left[1+\frac{\psi^4}{180}-\frac{10\phi^2\psi^2}{180}+\frac{9\phi^4}{180}\right]\cdots$$

$$=\frac{\phi}{\psi}+\frac{\phi}{\psi}\cdot\tfrac{1}{180}[\phi^2-\psi^2][9\phi^2-\psi^2].$$

In Mr. Greenwood's solution it will be noticed that $\frac{\tan\phi}{\tan\psi}$ is *not AB* as stated.

The relation $\tan\phi=\phi+\frac{1}{3}\phi^3+\frac{2}{15}\phi^5+\frac{17}{315}\phi^7\ldots$ is readily obtained from

$$\tan\phi=\left[\phi-\frac{\phi^3}{6}+\frac{\phi^5}{120}-\frac{\phi^7}{5040}+\ldots.\right]\left[1-\frac{\phi^2}{2}+\frac{\phi^4}{24}-\frac{\phi^6}{720}+\ldots.\right]^{-1}.$$

$$\text{The second factor}=\left[1+\left(\frac{\phi^2}{2}-\frac{\phi^4}{24}+\frac{\phi^6}{720}\right)+\left(\frac{\phi^2}{2}-\frac{\phi^4}{24}\right)^2+\left(\frac{\phi^2}{2}\right)^3+\ldots.\right]$$

and on multiplying out the expansion is obtained to any power we require.

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PROBLEMS FOR SOLUTION.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

157. Proposed by A. H. HOLMES, Brunswick, Maine.

Find integral values for m and n in $64m^2n^2(m^2-n^2)^2+(m^2+n^2)^4=\square$.

158. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Find positive rational values of a and b in the equation $x^4-2ax^2+x+a^2-b=0$, that will make each of the roots (all different) rational numbers.